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The Optimum Plate Aspect Ratio for the Best Performance in a Flat-Plate Thermal Diffusion Column with Transverse Sampling Streams

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ABSTRACT

The effects of plate aspect ratio on the degree of separation, production rate, and plate surface area in a flat-plate thermal diffusion column with transverse sampling streams have been investigated. Theoretical considerations show that when a thermal diffusion column is constructed with the best plate aspect ratio, either maximum separation or maximum production rate or minimum plate surface area can be obtained. The optimum plate aspect ratio for maximum separation is obtained with a given production rate and plate surface area, while that for the maximum production rate is determined with the degree of separation and plate surface area fixed, and that for the minimum plate surface area is estimated with a known degree of separation and the production rate. It is interesting that the optimum plate aspect ratio for maximum separation is exactly the same as that for minimum plate surface area. The maximum separation and maximum production rate are achieved without changing the total expenditure, while the design with minimum plate surface area results in minimizing the total expenditure.

INTRODUCTION

Thermal diffusion is an unusual process for separating a liquid or gas mixture (2, 6). Mixtures which are difficult or impossible to separate by distillation, extraction, or any other usual method may be successfully separated by thermal diffusion. It was used to separate the isotopes of uranium at Oak Ridge in World War II.

Thermal diffusion takes place when a temperature gradient of two gases or liquids gives rise to a concentration gradient with one component concentrated near the hot wall and the other component concentrated near the cold wall. Two methods of utilizing thermal diffusion for separation of solutions have been proposed. In the static method the thermal gradient is established in such a manner that convection is eliminated and there is no bulk flow. Since the concentration gradient at steady state is such that the flux due to ordinary diffusion just counterbalances that resulting from thermal diffusion, the extent of the separation obtainable by the static method is generally very slight and it is of theoretical interest only. The second method, the thermogravitational method, introduced by Clusius and Dickel (4, 5), multiplies the separation achieved in the static method by utilizing convection currents to produce a cascading effect.

Clusius and Dickel showed that a horizontal temperature gradient produces not only thermal diffusion in the direction of the temperature gradient but also natural convection of the fluid upward near the hot surface and downward near the cold surface. These convective currents produce a cascading effect analogous to the multistage effect of a countercurrent extraction, and as a result a considerably greater separation may be obtained. An excellent treatment of column theory was given by Furry, Jones, and Onsager (8).

A more detailed study of the mechanism of separation in the Clusius–Dickel column indicates that the convective currents, in addition to the desirable cascading effect; also produce an undesirable remixing effect, and these two effects conflict with one another. Convective currents have a multistage effect which is necessary to secure high separation, and it is an essential feature of the Clusius-Dickel column. However, since the convection brings down the fluid at the top of the column, where it is rich in one component, to the bottom of the column, where it is rich in the other component, and vice versa, there is a remixing of the two components. It therefore appears that proper control of the convective strength might effectively suppress this undesirable remixing effect while still preserving the desirable cascading effect, and thereby lead to improved separation. Based on this concept, some improved columns have been developed in the literature, including inclined columns (10), wired columns (13), packed columns (9, 12), inclined moving-wall columns (11), rotated wired columns (14) and barrier columns (16).

In developing these improved columns, a number of studies of the operating variables in the thermal diffusion column have been made. There is still an important term, the plate aspect ratio (the ratio of plate length to plate width), which affects separation efficiency and which has hardly ever been discussed. With a constant plate surface area, increasing the plate aspect ratio will increase the length of the separation section, which is good for separation. However, increasing the plate aspect ratio also decreases the plate width and thus increases the flow velocity, resulting in enhancement of the remixing effect, which is bad for separation. Therefore, the plate aspect ratio must be well designed.

In conventional thermogravitational thermal diffusion columns (the C-D column), the feed is introduced at the middle and the products are withdrawn from the top and bottom. In industrial applications, however, thermal diffusion columns are connected in series, such as in the Frazier scheme (7), and the feeding method in this scheme is such that the sampling streams do not pass through but move outside the columns. Figure 1 shows a single column of the Frazier scheme. It is the purpose of this work to investigate the effect of plate aspect ratio on the performance in a flat-plate thermal diffusion column with transverse sampling streams. The optimum plate aspect ratio for maximum separation and maximum production rate will be determined with constant plate surface area. Further, the optimum aspect ratio for a minimum plate surface area will be determined for fixed degree of separation and production rate.

SEPARATION THEORY

Consider a flat-plate thermal diffusion column with gap (2ω), length L, and width B, as shown in Fig. 1. The delivery of supplies σ with concentration C_0 are accomplished at the upper and lower ends of the column, and sampling of the products with the same flow rate σ is carried out at the ends opposite to the supply entrances.

Velocity Distribution

Since the space between the plate surfaces of the column is small, we may assume that the convective flow produced by the density gradient is laminar and that the temperature distribution is determined by conduction in the x-direction only. We also assume that the convection velocity is in the z-direction only, and that the mass fluxes due to thermal and ordinary diffusion are too small to affect the velocity and temperature profiles. Applying the appropriate equations of motion and energy gives the following steady-state velocity profile (1):

$$V_z = \frac{\rho\beta_{\bar{T}}\omega^2(\Delta T)g}{12\mu}(\eta - \eta^3) \tag{1}$$

where

$$\eta = x/\omega \tag{2}$$

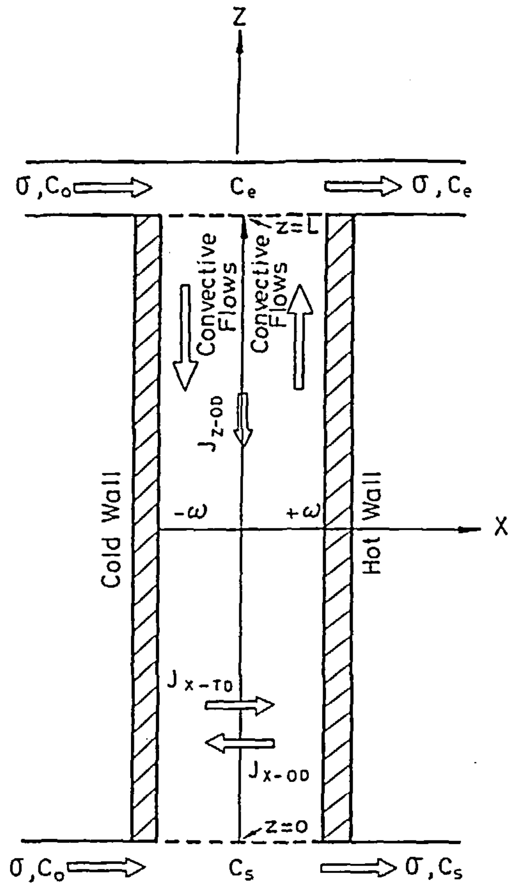

FIG. 1 Schematic diagram of the flat-plate thermal diffusion column with transverse sampling streams.

Concentration Distribution

The horizontal mass flux of component 1 of a binary mixture is related to the velocity by the differential mass balance equation

$$\frac{\partial J_x}{\partial \eta} + \rho\omega V_z \frac{\partial C}{\partial z} = 0 \tag{3}$$

Under the assumptions that diffusion in the z-direction and bulk flow in the x-direction are negligible, the expression for J_x in terms of the two contributions, ordinary and thermal diffusion, is (8)

$$J_x = \frac{D\rho}{\omega}\left(-\frac{\partial C}{\partial \eta} + \frac{\alpha C(1 - C)}{\overline{T}}\frac{dT}{d\eta}\right) \tag{4}$$

The concentration profile may be calculated by combining Eqs. (1), (3), and (4) and integrating with the following boundary conditions:

$$\text{at } \eta = \pm 1, \qquad J_x = 0 \tag{5}$$

$$\text{at } \eta = \eta, \qquad C = C(\eta, z) \tag{6}$$

$$\text{at } \eta = 0, \qquad C = C_z(0, z) \tag{7}$$

The solution is

$$C = C_z + \frac{\alpha A(\Delta T)}{2\overline{T}}\eta + \frac{\beta_{\overline{T}}\omega^4(\Delta T)g}{12D\mu}\left(\frac{\partial C}{\partial z}\right)\left(\frac{\eta^3}{6} - \frac{\eta^5}{20} - \frac{\eta}{4}\right) \tag{8}$$

In obtaining the above solution, $dT/d\eta = \Delta T/2$ (1) as obtained from the energy equations was used, and we assumed that $\partial C/\partial z$ was a function of z alone and that the quantity $C(1 - C)$ appearing in the thermal diffusion term was constant because C does not change significantly along the column, i.e.,

$$C(1 - C) = A \text{ (a constant)} \tag{9}$$

Transport Equation

The rate of mass transport of component 1 in the z-direction is given by

$$\tau = \int_{-\omega}^{+\omega} \rho C V_z B dx - \int_{-\omega}^{+\omega} \rho D \frac{\partial C}{\partial z} B dx \tag{10}$$

Combining Eqs. (1), (8), and (10), and neglecting the ordinary diffusion in the opposite direction of z-axis, gives

$$\tau = AH - K\frac{dC}{dz} \tag{11}$$

where

$$H = \frac{\alpha\beta_{\overline{T}}\rho g(2\omega)^3 B(\Delta T)^2}{6!\mu \overline{T}_{\overline{T}}^{\,2}} \tag{12}$$

$$K = \frac{\beta_{\overline{T}}^2\rho g^2(2\omega)^7 B(\Delta T)^2}{9!D\mu^2} \tag{13}$$

In Eq. (11), τ is the transport of component 1 in the positive z-direction under steady operation. Moreover, $\partial C/\partial z$ is replaced by dC/dz since C varies mainly along z at this stage.

Equations of Separation, Production Rate, and Column Length

Making material balance for the top and bottom parts of the column, one obtains, respectively

$$\tau = AH - K\frac{dC}{dz} \tag{14}$$

$$\begin{aligned} &= \sigma(C_e - C_0) \\ &= \sigma(C_0 - C_s) \end{aligned} \tag{15}$$

The integration of Eqs. (14) and (15) from the top to the bottom of the column, which satisfies the boundary conditions:

$$\text{at } z = 0, \qquad C = C_s \tag{16}$$

$$\text{at } z = L, \qquad C = C_e \tag{17}$$

are

$$C_e - C_s = \left[A - \frac{\sigma}{H}(C_e - C_0)\right]\frac{HL}{K} \tag{18}$$

$$C_e - C_s = \left[A - \frac{\sigma}{H}(C_0 - C_s)\right]\frac{HL}{K} \tag{19}$$

Addition of Eqs. (18) and (19) gives the equation for calculating the degree of separation:

$$\Delta = C_e - C_s \tag{20}$$

$$= \frac{A(HL/K)}{1 + (\sigma L/2K)} \tag{21}$$

Furry et al. (8) pointed out that for equifraction solution ($0.3 < C < 0.7$), $A \approx 0.25$. Accordingly, Eq. (21) becomes

$$\overline{\Delta} = \frac{(HL/4K)}{1 + (\sigma L/2K)}, \qquad 0.3 < C < 0.7 \tag{22}$$

For the whole range of concentration ($0 \leq C \leq 1.0$), the method of least squares may be carried out with Eq. (9) by finding the appropriate choice of constant A for the functional

$$\min E = \int_{C_s}^{C_e} [C(1 - C) - A]^2 dC$$

to have a minimum (15), or simply

$$A = \frac{1}{C_e - C_s} \int_{C_s}^{C_e} C(1 - C)dC \tag{23}$$

In performing the integration of Eq. (23), the upper and lower limits are substituted by Eqs. (24) and (25):

$$C_e = C_0 + \Delta/2 \tag{24}$$

$$C_s = C_0 - \Delta/2 \tag{25}$$

The result is

$$A = C_0(1 - C_0) - \Delta^2/12 \tag{26}$$

Equations (24) and (25) were obtained from Eqs. (14), (15), and (20). The equation of separation for the whole range of concentration is then obtained by substituting Eq. (26) into Eq. (21) and using Eq. (22):

$$\Delta = \left[\left(\frac{1.5}{\overline{\Delta}}\right)^2 + 12C_0(1 - C_0)\right]^{1/2} - \frac{1.5}{\overline{\Delta}} \tag{27}$$

Note that the expression for $\overline{\Delta}$ in term of Δ can be obtained from Eq. (27) as

$$\overline{\Delta} = \frac{3\Delta}{12C_0(1 - C_0) - \Delta^2} \tag{28}$$

Since the parallel flat plates are rectangular, the plate surface area S is the product of plate length L and plate width B, i.e.,

$$S = LB \tag{29}$$

If we let ξ be the plate aspect ratio defined by

$$\xi = L/B \tag{30}$$

then, from Eqs. (29) and (30), one obtains

$$L = (S\xi)^{1/2} \tag{31}$$

$$B = (S/\xi)^{1/2} \tag{32}$$

and Eq. (22) becomes

$$\bar{\Delta} = \frac{(a/4b)(S\xi)^{1/2}}{1 + (\sigma\xi/2b)} \tag{33}$$

where

$$a = H/B = \frac{\alpha\rho\beta_T g(2\omega)^3(\Delta T)^2}{6!\mu\bar{T}} \tag{34}$$

$$b = K/B = \frac{\rho\beta_T^2 g^2(2\omega)^7(\Delta T)^2}{9!D\mu^2} \tag{35}$$

Equation (33) can be rewritten to obtain the expressions for calculating the production rate and plate surface area as

$$\sigma = (aS^{1/2}/2\bar{\Delta})/\xi^{1/2} - 2b/\xi \tag{36}$$

$$S = [4b\bar{\Delta}(1 + \sigma\xi/2b)/a]^2/\xi \tag{37}$$

MAXIMUM SEPARATION WITH PLATE SURFACE AREA FIXED

With constant plate surface area S, the optimum plate aspect ratio ξ^*_Δ for maximum separation Δ_{max} is obtained by partially differentiating Eq. (27) with respect to ξ and setting $\partial\Delta/\partial\xi = 0$. After differentiation and simplification, we obtain $\partial\bar{\Delta}/\partial\xi = 0$. This means that the optimum plate aspect ratio is independent of the solution concentration. Accordingly, partially differentiating Eq. (33) with respect to ξ and setting $\partial\bar{\Delta}/\partial\xi = 0$, we have

$$\xi^*_\Delta = (L/B)^*_\Delta = 2b/\sigma \tag{38}$$

From Eqs. (29)–(32) we also obtain

$$L^*_\Delta = S/B^*_\Delta = \sqrt{2}(bS/\sigma)^{1/2} \tag{39}$$

$$B^*_\Delta = S/L^*_\Delta = (1/\sqrt{2})(\sigma S/b)^{1/2} \tag{40}$$

Consequently, the maximum separation may be obtained from Eqs. (27) and (33) by the substitution of Eq. (38). The result is

$$\Delta_{max} = \left[\left(\frac{1.5}{\bar{\Delta}_{max}}\right)^2 + 12C_0(1 - C_0)\right]^{1/2} - \frac{1.5}{\bar{\Delta}_{max}} \tag{41}$$

where

$$\bar{\Delta}_{max} = 0.125(2a^2S/\sigma b)^{1/2} \tag{42}$$

Equations (38) and (42) show that whereas $\overline{\Delta}_{max}$ as well as Δ_{max} depends on the thermal diffusion constant α, $\xi_{\overline{\Delta}}^*$ is independent of α. The problem of finding the maximum separation Δ_{max} and the best plate aspect ratio $\xi_{\overline{\Delta}}^*$ for a specified flow rate σ can readily be solved by using Eqs. (38), (41), and (42) since a, b and S are constants for a given column and system. This problem, however, is rather artificial and academic in nature, and therefore two other more practical problems will be discussed: 1) finding the maximum production rate σ_{max} and the corresponding best plate aspect ratio for a given column operated in a manner to obtain predetermined degree of separation and plate surface area, and 2) finding the minimum plate surface area and the corresponding best plate aspect ratio required to obtain a specific degree of separation and production rate.

MAXIMUM PRODUCTION RATE WITH PLATE SURFACE AREA FIXED

The optimum plate aspect ratio for maximum production rate with the degree of separation Δ (as well as $\overline{\Delta}$) and plate surface area specified is obtained by partially differentiating Eq. (36) with respect to ξ and setting $\partial\sigma/\partial\xi = 0$. After differentiation and simplification, this gives

$$\xi_\sigma^* = (L/B)_\sigma^* = 64b^2\overline{\Delta}^2/a^2S \tag{43}$$

Therefore,

$$L_\sigma^* = (S/B_\sigma^*) = (S\xi_\sigma^*)^{1/2} = 8b\overline{\Delta}/a \tag{44}$$

$$B_\sigma^* = (S/L_\sigma^*) = (S/\xi_\sigma^*)^{1/2} = aS/8b\overline{\Delta} \tag{45}$$

Consequently, the maximum production rate may be obtained from Eq. (36) by substitution of Eq. (43). The result is

$$\sigma_{max} = \frac{a^2S}{32b\overline{\Delta}^2} \tag{46}$$

MINIMUM PLATE SURFACE AREA

The optimum plate aspect ratio for minimum plate surface area with the degree of separation Δ (as well as $\overline{\Delta}$) and flow rate σ specified is obtained by partially differentiating Eq. (37) with respect to ξ and setting $\partial S/\partial\xi = 0$. After differentiation and simplification, one obtains

$$\xi_S^* = (L/B)_S^* = 2b/\sigma \tag{47}$$

TABLE 1

Comparison of Separations Δ_{max} and Δ Obtainable at the Best Plate Aspect Ratio ξ^*_Δ and at $\xi = 18.5$, respectively, with $S = 0.185$ m^2

						$C_0 = 0.1$ or 0.9		$C_0 = 0.3$ or 0.7		$C_0 = 0.5$	
$\sigma \times 10^5$ (kg·s^{-1})	$\bar{\Delta}$ (%)	ξ^*_Δ (m)	L^*_Δ (m)	B^*_Δ (m)	$\bar{\Delta}_{max}$ (%)	Δ (%)	Δ_{max} (%)	Δ (%)	Δ_{max} (%)	Δ (%)	Δ_{max} (%)
0.817	8.40	170.9	6.623	0.0330	14.10	3.02	5.06	7.04	11.78	8.40	14.00
1.634	7.63	85.4	3.975	0.0465	9.97	2.74	3.58	6.40	8.35	7.63	9.94
3.268	6.47	42.7	2.811	0.0658	7.05	2.33	2.54	5.43	5.91	6.47	7.04
6.536	4.97	21.4	1.990	0.0930	4.98	1.79	1.79	4.17	4.18	4.97	4.98
(7.546)	(4.64)	(18.5)	(1.850)	(0.1000)	(4.64)	(1.67)	(1.67)	(3.90)	(3.90)	(4.64)	(4.64)
13.072	3.40	10.7	1.407	0.1315	3.52	1.22	1.27	2.86	2.96	3.40	3.52
26.144	2.08	5.3	0.990	0.1868	2.49	0.75	0.90	1.75	2.09	2.08	2.49

Therefore,

$$L^*_S = (S_{min}/B^*_S) = (S_{min}\xi^*_S)^{1/2} = 8b\bar{\Delta}/a \tag{48}$$

$$B^*_S = (S_{min}/L^*_S) = (S_{min}/\xi^*_S)^{1/2} = 4\bar{\Delta}\sigma/a \tag{49}$$

$$S_{min} = 32\bar{\Delta}^2 b\sigma/a^2 \tag{50}$$

NUMERICAL EXAMPLES

The improvement in performance resulting from operating at the best plate aspect ratio may be illustrated by using the experimental data of

TABLE 2

Comparison of Production Rates σ_{max} and σ Obtainable at the Best Plate Aspect Ratio ξ^*_σ and at $\xi = 18.5$, respectively, with $S = 0.185$ m^2

Δ (%)								
$C_0 = 0.1$ or 0.9	$C_0 = 0.3$ or 0.7	$C_0 = 0.5$	$\bar{\Delta}$ (%)	$\sigma \times 10^5$ (kg·s^{-1})	ξ^*_σ	L^*_σ (m)	B^*_σ (m)	$\sigma_{max} \times 10^5$ (kg·s^{-1})
3.02	7.04	8.40	8.40	0.817	60.7	3.351	0.055	2.301
2.74	6.04	7.63	7.63	1.634	50.1	3.044	0.061	2.789
2.33	5.43	6.47	6.47	3.268	36.0	2.581	0.072	3.878
1.79	4.17	4.97	4.97	6.536	21.2	1.980	0.093	6.572
(1.67)	(3.90)	(4.64)	(4.64)	(7.546)	(18.5)	(1.850)	(0.100)	(7.546)
1.22	2.86	3.40	3.40	13.072	9.9	1.353	0.137	14.043
0.75	1.75	2.08	2.08	26.144	3.7	0.827	0.227	37.523

TABLE 3
Minimum Plate Surface Area S_{min} Obtainable at the Best Plate Aspect Ratio ξ_S^*. $S = 0.185$ m^2 at $\xi = 18.5$

Δ (%)								
$C_0 = 0.1$ or 0.9	$C_0 = 0.3$ or 0.7	$C_0 = 0.5$	$\bar{\Delta}$ (%)	$\sigma \times 10^5$ (kg·s^{-1})	ξ_S^*	L_S^* (m)	B_S^* (m)	S_{min} (m^2)
3.02	7.04	8.40	8.40	0.817	170.9	3.358	0.020	0.066
2.74	6.04	7.63	7.63	1.634	85.4	3.038	0.036	0.108
2.33	5.43	6.47	6.47	3.268	42.7	2.581	0.060	0.156
1.79	4.17	4.97	4.97	6.536	21.4	2.273	0.081	0.184
(1.67)	(3.90)	(4.64)	(4.64)	(7.546)	(18.5)	2.359	0.078	(0.185)
1.22	2.86	3.40	3.40	13.072	10.7	1.355	0.127	0.172
0.75	1.75	2.08	2.08	26.144	5.3	0.830	0.155	0.129

Chueh and Yeh's work (3). The conditions are: benzene and n-heptane system; $\Delta T = 164 - 95 = 69°F = 38.3°C$; $B = 0.1$ m; $L = 1.85$ m; $S = 0.185$ m^2; $H = 0.845$ g/min; $K = 419$ g·cm/min. The values of constants a and b for this experiment are determined from Eqs. (34) and (35). They are

$$a = H/B = 0.845/[(0.1)(60)(1000)] = 1.4 \times 10^{-4} \text{ kg·s}^{-1}\text{·m}^{-1}$$

$$b = K/B = 419/[(0.1)(60)(1000)(100)] = 6.98 \times 10^{-4} \text{ kg·s}^{-1}$$

Consequently, the maximum separation Δ_{max}, the maximum production rate σ_{max}, as well as their corresponding best plate aspect ratio were calculated from the appropriate equations with the plate surface area S equal to 0.185 m^2. The results are presented in Tables 1 and 2, respectively. The minimum plate surface area S_{min} and its corresponding best plate aspect ratio were also calculated with given separation Δ and production rate σ. These results are presented in Table 3.

DISCUSSIONS AND CONCLUSION

The solutions of optimum plate aspect ratio for maximum separation, maximum production rate, and minimum plate surface area have been obtained as shown by Eqs. (38) and (42), by Eqs. (43) and (46), and by Eqs. (47) and (50), respectively. It was found that all the best plate aspect ratios for Δ_{max}, σ_{max}, and S_{min} are independent of feed concentration C_0. The improvement in performance was illustrated numerically by using the experimental data of the benzene and n-heptane system (3), and the results are presented in Tables 1, 2, and 3.

The comparison of separations Δ_{max} and Δ, obtainable at the best corresponding plate aspect ratio ξ_Δ^* and at $\xi = (L/B) = 18.5$, respectively, under various flow rates σ and feed concentrations C_0 with constant plate surface area ($S = 0.185$ m^2), is shown in Table 1. It is seen from this table that the best plate aspect ratio for maximum separation decreases as the flow rate increases. An improvement in separation is really obtained, especially for ξ_Δ^* far from 18.5. When $\sigma = 7.546$ kg·s^{-1}, $\xi_\Delta^* = \xi = 18.5$, and $\Delta_{max} = \Delta$, the Chueh and Yeh's experimental system operating at this flow rate is exactly the case where the plate aspect ratio is optimum and the separation is maximum.

Comparison of production rates σ_{max} and σ obtained at the best corresponding plate aspect ratio ξ_σ^* and at $\xi = 18.5$, respectively, under various feed concentrations and degree of separation Δ with $S = 0.185$ m^2, is presented in Table 2. It is shown in Table 2 that the best plate aspect ratio for the maximum production rate increases when the specified degree of separation increases, and that the improvement in production rate is really obtained when ξ_σ^* is far from 18.5 It is also found that when $\bar{\Delta} = 0.0464$, $\xi_\sigma^* = \xi = 18.5$, and $\sigma_{max} = \sigma = 7.546 \times 10^{-5}$ kg·s^{-1}, Chueh and Yeh's experimental system operating for this specific degree of separation ($\bar{\Delta} = 0.0464$) is exactly the case where the plate aspect ratio is optimum and the production rate is maximum.

Table 3 shows the minimum plate surface area S_{min} and the corresponding best plate aspect ratio ξ_S^* under various flow rates, feed concentrations, and specified degree of separation. An interesting result obtained is that the optimum plate aspect ratio for maximum separation is exactly the same as that for the minimum plate surface area, as shown by Eqs. (38) and (47), or in Tables 1 and 3. It is observed from Table 3 that when $\sigma = 7.546 \times 10^{-5}$ kg·s^{-1}, and $\bar{\Delta} = 0.0464$, $S_{min} = S = 0.185$ m^2, and thus Chueh and Yeh's experimental system operating at these required conditions is exactly the case where the plate aspect ratio is optimum and the plate surface area is minimum.

The expenditure involved in making a separation by thermal diffusion essentially includes two parts: a fixed charge and an operating expense. The fixed charge is roughly proportional to the equipment cost, say the plate surface area, $S = BL$, while the operating expense is mainly heat. The heat transfer rate is also proportional to the plate surface area (heat transfer area) if $\Delta T/2\omega$ or both ΔT and 2ω are specified. Therefore, the total expenditure is almost fixed as long as the plate surface area is kept unchanged. Since the maximum separation and maximum production rate are obtained with a constant plate surface area, the values of Δ_{max} and σ_{max} in Tables 1 and 2, respectively, were calculated with the total expenditure fixed. On the other hand, the use of a minimum plate surface area will minimize both the fixed charge and the operating expense, and therefore the values of S_{min} and ξ_ξ^* in Table 3 provide the optimum design of minimum total expenditure with specified separation and production rates.

SYMBOLS

A	constant defined by Eq. (9)
a	system constant defined by Eq. (34) ($kg \cdot m^{-1} \cdot s^{-1}$)
B	plate width (m)
B_Δ^*, B_σ^*, B_ξ^*	optimum value of B (m)
b	system constant defined by Eq. (35) ($kg \cdot s^{-1}$)
C	fractional mass concentration of component 1
C_s, C_e	C in the product stream existing from the stripping, enriching section
C_0	C in the feed streams
$C_z(0, z)$	C at $x = 0$ in the column
D	ordinary diffusion coefficient ($m^2 \cdot s^{-1}$)
g	gravitational acceleration ($m \cdot s^{-2}$)
H	system constant defined by Eq. (12) ($kg \cdot s^{-1}$)
$J_{x\text{-OD}}$, $J_{x\text{-TD}}$	mass flux of component 1 in x-direction due to ordinary, thermal diffusion ($kg \cdot m^{-2} \cdot s^{-1}$)
$J_{z\text{-OD}}$	mass flux of component 1 in z-direction due to ordinary diffusion ($kg \cdot m^{-2} \cdot s^{-1}$)
K	system constant defined by Eq. (13) ($kg \cdot m \cdot s^{-1}$)
L	plate length (m)
L_Δ^*, L_σ^*, L_ξ^*	optimum value of L (m)
S	BL, plate surface area (m^2)
S_{min}	minimum value of S (m^2)
$\overline{T}$	reference temperature (K)
ΔT	difference in temperature of hot and cold surfaces (K)
V_z	velocity profile ($m \cdot s^{-1}$)
x	axis normal to the plate surface (m)
z	axis parallel to the flowing direction (m)

Greek Letters

α	thermal diffusion constant
$\beta_{\overline{T}}$	$-(1/\rho)(\partial\rho/\partial T)_P$ evaluated at $\overline{T}$ (K^{-1})
Δ	$C_e - C_s$
$\overline{\Delta}$	Δ obtained when $0.3 < C < 0.7$
Δ_{max}, $\overline{\Delta}_{max}$	maximum value of Δ, $\overline{\Delta}$
μ	viscosity ($kg \cdot m^{-1} \cdot s^{-1}$)

ξ	L/B, plate aspect ratio
ξ_Δ^*, ξ_σ^*, ξ_S^*	optimum value of ξ
ρ	mass density (kg·m^{-3})
σ	mass flow rate (kg·s^{-1})
σ_{max}	maximum value of σ (kg·s^{-1})
τ	transport of component 1 along z-direction (kg·s^{-1})
ω	one-half of the plate spacing of the column (m)

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